Midterm 1 / 2003.3.12 / Theory of Functions of a Complex Variable II / MAT 5233.001

Name: ______________________________

Please show all work.

1. (10 pts.) Prove the first part of the Weierstrass theorem: Suppose Ω is a domain in the complex plane and f_n is a sequence in $\mathcal{H}(\Omega)$ such that $f_n \to f$ uniformly on compact subsets of Ω. Prove that $f \in \mathcal{H}(\Omega)$.

2. (10 pts.) Find the first three nontrivial terms of the Laurent series at the origin of

   (a) $f(z) = e^z \sin(3z^2)$

   (b) $f(z) = 1/\sin(z)$

3. (10 pts.) Prove that a nonconstant entire function must have a dense image.

4. (10 pts.) Let $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$. For each of the following covering maps p, how many elements are there in each stalk $p^{-1}(x)$? Compute and sketch $p^{-1}(i)$. Illustrate that p is indeed a covering map by sketching an evenly covered neighborhood of i and its preimage under p.

   (a) $p(z) = z^2 \colon \mathbf{C}^* \to \mathbf{C}^*$

   (b) $p(z) = e^z \colon \mathbf{C} \to \mathbf{C}^*$

1	2	3	4	total (40)
				%

Prelim. course grade: %