Theory of Functions of a Complex Variable II / MAT 5233
Midterm 1 / March 10, 1999 / Instructor: D. Gokhman

Name: _____

Please show all work.

1. (16 pts.) Find and classify all singularities for the following functions:

$$\text{(a) } f(z) = \frac{\cos z}{z} \quad \text{(b) } f(z) = z^2 \sin \frac{1}{z} \quad \text{(c) } f(z) = \frac{z}{z^2 - 1} \quad \text{(d) } f(z) = \tan z$$

2. (10 pts.) How many zeros (counting multiplicities) does $p(z) = z^8 + 3z - 1$ have in the annulus $\{z \in \mathbf{C}\colon 1 < |z| < 2\}$?

3. (10 pts.) For $f(z)$ in #1c find a Laurent expansion valid in $\{z \in \mathbf{C}\colon 1 < |z|\}$.

4. (10 pts.) Suppose f is entire and $|f(x + iy)| \leq e^x$ on the unit circle. Show that this relation holds on the unit disc.

1	2	3	4	total (46)	%