Theory of Functions of a Complex Variable II, MAT 5233
Final, due 19:45 Monday, May 8, 1995
Instructor: D. Gokhman

Name: ______________________________

1	2	3	4	5	6	total

1. SERIES

   1. (Hille 5.5.6) What is the sum of the series $\sum_{n=1}^{\infty} n^2 z^n$? What is the radius of convergence?
   2. (Hille 5.6.2) Express $\sum_{n=0}^{\infty} P(n) z^n / n!$ in terms of e^z, if $P(n) = a_0 + a_1 n + a_2 n^2$. What is the radius of convergence?
   3. Consider the series $\sum_{n \in \mathbf{Z} \setminus \{0\}} e^{nz}/n$. Find the set of convergence of the series. Where is the convergence uniform? What function does the series converge to? Include proofs (you may refer to theorems).

2. RESIDUES

   1. (Hille 9.1.1d,e, B/G 1.12.3) Find $\mathrm{Res}_\omega S$ for the idicated 1-forms ω and subsets $S \subseteq \mathbf{C}$:

   $$\text{(a) } \omega = \frac{(z^4+1)\,dz}{z^2(z-2)^3}, \quad S = \{2\} \qquad \text{(b) } \omega = \frac{dz}{z^2 \sin z}, \quad S = \{0\}$$

   $$\text{(c) } \omega = dz/z, \quad S = \overline{B}(0,1) \qquad \text{(d) } \omega = dz/z, \quad S = \overline{B}(3,1)$$

   2. (B/G 1.12.4) Let $T_j, j = 1, ...n$ be holes of a domain $\Omega \subseteq \mathbf{C}$ and $\lambda_j, j = 1, ...n$ arbitrary complex numbers. Construct a closed 1-form ω on Ω such that $\mathrm{Res}_\omega T_j = \lambda_j$.
   3. (Hille 9.1.5c) Use residues to calculate the improper real integral:

   $$\int_{-\infty}^{\infty} \frac{x^2-1}{(x^2+1)^2}\,dx$$