Midterm 2 / 2002.11.25 / Theory of Functions of a Complex Variable I / MAT 5223.001

Name: _____

Please show all work and justify your answers.

1. (10 pts.) Evaluate the following integrals along the given paths (sketch):

   (a) $\displaystyle\int_\gamma \frac{dz}{z^3 - 2iz^2}$, where $\gamma$ is the unit circle

   (b) $\displaystyle\int_\gamma \overline{z}\, dz$, where $\gamma = \{z : |z - 1 + i| = 1\}$

2. (10 pts.) Let $\displaystyle I(r) = \int_\gamma \frac{1}{z^5 + 1}\, dz$, where $\gamma$ is the top half of a circle centered at the origin of radius $r > 1$. Show that $I(r) \to 0$ as $r \to \infty$.

3. (10 pts.) Suppose $f_n$ is a sequence of continuous functions on a domain $\Omega$ and $f_n \to f$ uniformly on compact subsets of $\Omega$. Prove that for any rectifiable path $\gamma$ in $\Omega$

$$\int_\gamma f_n(z)\, dz \to \int_\gamma f(z)\, dz$$

4. (10 pts.) Suppose $\Omega$ is a domain and $f \in \mathcal{H}(\Omega)$ is nonconstant. Show that a local minimum of $|f|$ can occur in $\Omega$ only at a root of $f$.

5. (10 pts.) Suppose $f$ is entire and $|f(z)| \leq |z|$ for all $z$ with $|z| > 1$. Prove that $f$ is a polynomial of degree at most 1.

| 1 | 2 | 3 | 4 | 5 | total (50) |
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Prelim. course grade: %