Midterm 1 / 2002.10.16 / Theory of Functions of a Complex Variable / MAT 5223.001

Name: ____

Please show all work.

1. (10 pts.) Construct a fractional linear transformation that takes the upper half plane to the disc of radius 1 centered at -1. Is such a fractional linear transformation unique? Explain.

2. (10 pts.) Suppose $f: \mathbf{C} \to \mathbf{C}$ is entire (analytic everywhere). Prove the following:

   (a) If $\overline{f(z)}$ is entire, then $f(z) = \text{const}$.

   (b) If $f(\overline{z})$ is entire, then $f(z) = \text{const}$.

3. (10 pts.) Let $u \neq 0$ and define $f: \mathbf{C} \to \mathbf{C}$ by $f(z) = uz$. Prove that f is conformal at 0.

   Hints: Pick arbitrary nonzero z and w and show that the angle between $f(z)$ and $f(w)$ is the same as the angle between z and w. Use either polar coordinates or linear algebra.

4. (10 pts.) Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(z+i)^n}{2^n n^2}$.

   (a) Find the radius of convergence and sketch the disc of convergence.

   (b) Prove that the series converges on the boundary of the disc of convergence.

1	2	3	4	total (40)
				%

Prelim. course grade: %