Theory of Functions of a Complex Variable I, MAT 5223
Midterm, October 21, 1996
Instructor: D. Gokhman

Name: ____________________

Show all work. Box your answers.

1. (20 pts.) Find and sketch all $z \in \mathbf{C}$ such that

   (a) $z^5 - 1 + i = 0$ (b) $1 + z + z^2 + z^3 + z^4 = 0$

2. (24 pts.) For each of the following sets $E \subseteq \mathbf{C}$ find the limit set $E'$. Sketch both $E$ and $E'$.

   (a) $E = \{i^n \colon n \in \mathbf{Z}\}$ (b) $E = \{z \in \mathbf{C} \colon 0 < |z| < 1\}$ (c) $E = \left\{e^{i\theta} \in \mathbf{C} \colon \theta \in \mathbf{Q}\right\}$

3. (40 pts.) For the following functions $f\colon \mathbf{C} \to \mathbf{C}$ find the largest subset of $\mathbf{C}$, where $f$ is $\mathbf{C}$-differentiable.

   (a) $f(z) = z$ (b) $f(z) = \overline{z}$ (c) $f(z) = z\overline{z}$ (d) $f(z) = e^{-z}$

4. (20 pts.) Find a Möbius transformation which takes the outside of the circle of radius 2 centered at $i$ to the upper half plane $\{z \in \mathbf{C} \colon \operatorname{Im} z > 0\}$.

5. (28 pts.) True or false — circle your choice. No justification necessary.

T F (a) The group of Möbius transformations is commutative.
T F (b) Stereographic projection is conformal.
T F (c) Each complex polynomial can be factored completely.
T F (d) If $z_n \to \infty$ then $\operatorname{Re} z_n \to \infty$ and $\operatorname{Im} z_n \to \infty$.
T F (e) If $z_n \to z$, then $|z_n| \to |z|$ and $\operatorname{Arg} z_n \to \operatorname{Arg} z$.
T F (f) If $f$ and $g$ are entire maps $\mathbf{C} \to \mathbf{C}$, then so is $f \circ g$.
T F (g) If $f$ is $\mathbf{C}$-differentiable, then $f$ is continuous.

| 1 | 2 | 3 | 4 | 5 | total |
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