<u>University of Texas at San Antonio</u>

Complex Variable I, MAT 5223
Exam №1, 10/19/92
Instructor: D. Gokhman

Name: ______________________________

1. (30 pts.)
   (a) Find all $\theta \in [0, 2\pi)$ such that $\left|e^{i\theta} - 1\right| = 2$.
   (b) Find all $z \in \mathbf{C}$ such that $1 + z + z^2 + ... + z^n = 0$.
   (c) Find all $z \in \mathbf{C}$ such that $\operatorname{Re} z^n \geq 0 \ \forall n \in \mathbf{N}$.

2. (30 pts.) Suppose $E \subseteq \mathbf{C}$ and
   E' is the set of all limit points of E.
   (a) Prove that $\overset{\circ}{E} \subseteq E'$.
   (b) Find E' for $E = \left\{i^k \colon k \in \mathbf{N}\right\}$.
   (c) Find E' for $E = \left\{z \colon \exists k \in \mathbf{N} \ z^k = 2\right\}$.

3. (40 pts.) True of false questions, circle your choice. If you choose TRUE, prove the assertion somewhere in the lower part of the page. If FALSE, provide a counterexample.

   Suppose X is a metric space and $f : X \to \mathbf{C}$ is continuous.

T F (a) If $E \subseteq \mathbf{C}$ is closed, then $f^{-1}(E)$ is closed.

T F (b) If $F \subseteq X$ is closed, then $f(F)$ is complete.

T F (c) If X is compact, then f is Lipshitz continuous.

T F (d) If X is path connected, then so is $f(X)$.