

Name: _____

Please show all work and justify your answers.

1. Determine whether $A = \{1/n : n = 1, 2, \dots\}$ is an open, closed, both, or neither as a subset of the real line. Prove your assertion.
2. Suppose X and Y are topological spaces and $f: X \rightarrow Y$. Prove that
 - (a) if $U \subseteq Y$, then $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$
 - (b) f is continuous on $X \Leftrightarrow \forall$ closed $V \subseteq Y, f^{-1}(V)$ is closed in X
3. Suppose X is a topological space and $A \subseteq X$. Prove that the boundary of A is the intersection of the closures of A and its complement in X .
4. Let A be the interval $[0, 1) \subseteq \mathbf{R}$ with the subspace topology.
 - (a) Explain why A is not compact.
 - (b) Prove it directly by exhibiting an open cover of A that has no finite subcover.

1	2	3	4	total (40)