

Name: _____

Please show all work and justify your statements.

1. Prove that $n^3 - n$ is divisible by 6 for any natural number n .
Hint: induction.
2. Suppose $\gcd(a, b) = p$, where p is a prime. What are the possible values for $\gcd(a^2, b)$? Explain.
3. Prove that $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$ is an integer for any integer n .
Hint: bring to a common denominator and compute the numerator modulo 3 and 5.
4. Solve $18x \equiv 12 \pmod{14}$.
5. Solve the system $x \equiv 3 \pmod{5}$, $x \equiv 1 \pmod{4}$, $x \equiv 2 \pmod{3}$.
6. Solve $x^3 - 2x^2 \equiv 2 \pmod{25}$.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %