Name: ____

Please show all work.

1. Sketch the subgroup lattice for $\mathbf{Z}_{20}$. For each subgroup, list all the elements and indicate all possible generators of the subgroup.

2. Suppose G is a group and $a \in G$ such that $|a| = 13$. Prove there exists $b \in G$ such that $a = b^9$

3. Suppose $\alpha = (4,3,7,8,9)(1,3,7,5,2)(2,7,6)$ is a permutation in cycle notation.
   (a) Express α as a product of disjoint cycles.
   (b) Find the order of α. Explain.
   (c) Find the parity of α. Explain.
   (d) Simplify α^{659}

4. Prove that $5\mathbf{Z}/40\mathbf{Z}$ is isomorphic to $\mathbf{Z}_8$

5. Solve the following system of two congruence equations

$$4x \equiv 10 \bmod 13$$

$$6x \equiv 9 \bmod 11$$

   Hint: first separately solve each congruence for x

6. (a) How many group homomorphisms are there from $\mathbf{Z}$ to $\mathbf{Z}_9 \times \mathbf{Z}_{25}$? Explain.
   (b) How many of these are surjective? Explain.
   (c) How many of these are injective? Explain.

1	2	3	4	5	6	total (60)