Final exam / 2019.5.9 / MAT 4233.001 / Modern Abstract Algebra

Name: ______________________________

Please show all work.

1. Sketch the subgroup lattice for $\mathbf{Z}_{20}$. For each subgroup, list all the elements and indicate all possible generators of the subgroup.

2. Suppose $\alpha = (4,3,7,8,9)(1,3,7,5,2)(2,7,6)$ is a permutation in cycle notation.

   (a) Express α as a product of disjoint cycles.

   (b) Find the order of α. Explain.

   (c) Find the parity of α. Explain.

   (d) Simplify α^{659}

3. (a) How many group homomorphisms are there from $\mathbf{Z}$ to $\mathbf{Z}_9 \times \mathbf{Z}_{25}$? Explain.

   (b) How many of these are surjective? Explain.

   (c) How many of these are injective? Explain.

4. Suppose R is a finite commutative ring with unity and $a \in R, a \neq 0$. Show that a is either a zero divisor or a unit (but not both).

5. Let A be the set of all polynomials in $\mathbf{Z}[x]$, whose coefficients are divisible by 3.

   (a) Show the A is an ideal of $\mathbf{Z}[x]$

   (b) Is A a maximal ideal of $\mathbf{Z}[x]$? Explain.

1	2	3	4	5	6	total (60)