1. Let $\alpha = (2,1,4,9,8)(4,7,6,5,1)(3,6,4) \in S_9$ (permutation in cycle notation)

   (a) What is the order of α? Explain.

   (b) What is the parity of α? Explain.

   (c) Express α^{1943} as a product of disjoint cycles. Explain.

2. Suppose $\varphi\colon G \to G'$ and $\psi\colon G' \to G''$ are homomorphisms of groups.

   (a) Prove that their composite $\psi\varphi\colon G \to G''$ is a homomorphism.

   (b) Prove that $\ker \varphi \triangleleft \ker \psi\varphi$

   (c) Assuming G is finite and φ, ψ are surjective, express the index of $\ker \varphi$ in $\ker \psi\varphi$ in terms of the orders of G', G'' and prove your assertion.

3. (a) Prove that $\mathbf{Z}_8$ is not an internal product of two proper subgroups.

   (b) Same for the dihedral group D_4

4. (a) How many group homomorphisms $\varphi\colon \mathbf{Z}_{45} \to \mathbf{Z}_{27}$ are there?

   (b) How many of these φ are injective and how many are surjective?

   (c) Prove your assertions.