Midterm 1 / 2019.10.10 / MAT 4233.001 / Modern Abstract Algebra

Name: ____

Please show all work. If you use a theorem, name it or state it.

1. Suppose m and n are natural numbers. Prove that
   (a) any common divisor of m and n divides $\gcd(m, n)$.
   (b) $\operatorname{lcm}(m, n)$ divides any common multiple of m and n.
2. Suppose H is a subgroup of $\mathbf{Z}$ that contains two distinct primes. Prove that $H = \mathbf{Z}$.
3. Sketch the subgroup lattice for $\mathbf{Z}_{18}$. For each subgroup, list all the elements and indicate all possible generators of the subgroup.
4. Consider the set of all complex cube roots of unity $H = \{z \in \mathbf{C} \colon z^3 = 1\}$
   (a) Show H is a subgroup of the multiplicative group of nonzero complex numbers $\mathbf{C}^*$.
   (b) How many elements does H have? List them.
5. With H as in the preceding problem, define a function $\varphi \colon \mathbf{Z} \to H$ by $\varphi(k) = e^{2k\pi i/3}$.
   (a) Prove that φ is a group homomorphism.
   (b) Is φ 1-1? Onto? Explain.

1	2	3	4	5	total (50)