Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Suppose X is a set, $s \in X$ and F is a field. Let R be the ring of all functions $X \to F$ with pointwise operations. Let $I = \{f \in R : f(s) = 0\}$. Prove that I is a maximal ideal of R.

2. Suppose R is as in the preceding problem and $t \in X$. Let $J = \{f \in R\colon f(s) = f(t) = 0\}$. Prove that if $t \neq s$, then J is an ideal of R which is not prime.

3. Find the quotient and remainder for $x^5 + 4x^3 + 2x^2 + 3$ divided by $x + 6$ in $\mathbf{Z}_7[x]$.

4. Suppose F is a field and $s \in F$. Let $I = \{f \in F[x]\colon f(s) = 0\}$. Use the division algorithm to prove that I is the ideal generated by $x - s$.

5. Let J be the ideal generated by x and 2 in $\mathbf{Z}[x]$. Prove that J is a maximal ideal.

1	2	3	4	5	total (50)