

Name: \_\_\_\_\_

Please show all work. If you use a theorem, name it or state it.

1. Let  $H = \{z \in \mathbf{C}: z^n = 1\}$ . Prove that  $H$  is a subgroup of  $\mathbf{C}^*$  isomorphic to  $\mathbf{Z}_n$ .
2. Suppose  $\alpha = (1, 6, 2, 5, 3)(2, 6)(4, 7, 3, 5, 1, 2)$  is a permutation (in cycle notation). What is the order of  $\alpha$ ? What is the parity of  $\alpha$ ? Simplify  $\alpha^{2017}$ .
3. Prove that the set of all even permutations in the symmetric group  $S_n$  is a normal subgroup of  $S_n$ . Exhibit a subgroup of  $S_3$  that is not normal. Explain.
4. How many group homomorphisms are there from  $\mathbf{Z}$  to  $\mathbf{Z}_{24}$ ? How many of them are one-to-one? How many of them are onto? For those that are onto, what is the kernel? Explain.
5. Suppose  $G$  is finite group of order  $n$  and  $a \in G$ . Prove that  $a^n = e$ . What can you conclude about the order of  $a$ , if  $n$  is prime? What can you conclude about groups of prime order?

1	2	3	4	5	total (50)