Midterm 2 / 2017.4.20 / MAT 4233.001 / Modern Abstract Algebra

Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Let $G = \mathrm{Gl}\,(n, \mathbf{R})$ be the multiplicative group of all invertible $n \times n$ real matrices, K a multiplicative subgroup of nonzero real numbers $\mathbf{R}^*$ and $H = \{A \in G\colon \det A \in K\}$. Prove that H is a normal subgroup of G.
2. Find (with proof) a group homomorphism on the symmetric group S_n of all permutations on n elements (to a suitable range), whose kernel is the alternating subgroup A_n of all even permutations.
3. Let R be the ring of all functions $\mathbf{R} \to \mathbf{R}$ with pointwise operations and let $a \in \mathbf{R}$. Prove that $I = \{f \in R\colon f(a) = 0\}$ is a maximal ideal of R.
4. With R as in the preceding problem, let $a \neq b \in \mathbf{R}$. Prove that $J = \{f \in R\colon f(a) = 0, f(b) = 0\}$ is an ideal of R that is not prime. Find (with proof) two prime ideals containing J.
5. Suppose F is a field and p is polynomial in $F[x]$ of degree 3. Prove that p irreducible if and only if p has no roots in F.

1	2	3	4	5	total (50)