Midterm 1 / 2017.2.23 / MAT 4233.001 / Modern Abstract Algebra

Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Let $m \in \mathbf{N}$ and $m\mathbf{Z} = \{mn\colon n \in \mathbf{Z}\}$. Prove $m\mathbf{Z} < \mathbf{Z}$. Conversely, prove that any subgroup of $\mathbf{Z}$ is of this form.

   Hint: given $H < \mathbf{Z}$, let m be the smallest positive element of H.

2. Suppose $\alpha = (1,2,3)(2,3,4,5)$ is a permutation (in cycle notation). What is the order of α? What is the parity of α? Express α^{2017} as a product of disjoint cycles.

3. Suppose G is finite group of order n and $a \in G$. Prove that $a^n = e$. What conclusions can you draw about the order of a, if $a \neq e$ and n is prime? What conclusion can you draw about groups of prime order?

4. Let $H = \{z \in \mathbf{C}\colon z^n = 1\}$. Prove that H is a subgroup of $\mathbf{C}^*$ isomorphic to $\mathbf{Z}_n$.

5. Prove $\mathrm{Aut}(\mathbf{Z}) \cong \mathbf{Z}_m$ $(m = ?)$

1	2	3	4	5	total (50)