Midterm 1 / 2015.10.1 / MAT 4233.001 / Modern Abstract Algebra

Name: _____

Please show all work and justify your answers.

1. For $a, b \in \mathbf{N}$ prove
   (a) any common divisor of a and b divides $\gcd(a, b)$.
   (b) $\operatorname{lcm}(a, b)$ divides any common multiple of a and b.
2. Sketch the subgroup lattices for $\mathbf{Z}_4$ and the dihedral group D_4.
3. Prove or disprove every group of prime order is cyclic.
4. If G is a group, $a \in G$, and $|a| = 8$, prove there exists $b \in G$ such that $b^3 = a$.
5. Suppose G is a group, $a \in G$, and $|a| = \left|a^2\right|$. What can you conclude about $|a|$? Make and prove a statement of the form $|a| = \left|a^2\right|$ if, and only if, $|a|$ is ...

1	2	3	4	5	total (50)