Final exam / 2015.12.7 / MAT 4233.001 / Modern Abstract Algebra

Name: ______________________________

Please show all work and justify your answers.

1. Suppose $H$ is a proper subgroup of $\mathbf{Z}$. Show that $H = \langle x \rangle$, where $x$ is the smallest positive element of $H$.

2. Sketch the subgroup lattice of the symmetric group $\Sigma_3$.

3. For each divisor $d$ of $|A_4|$, how many elements of order $d$ does $A_4$ have?

4. Let $\tau$ be the permutation $(1,4,5)(1,3,5,2)$. Factor $\tau^{2015}$ into disjoint cycles.

5. Exhibit two nontrivial proper subgroups of the dihedral group $D_4$, one that is normal in $D_4$ and one not. Prove your assertions.

6. Suppose $R$ is a commutative ring with unity and $a \in R$, $a \neq 0$. Define $\varphi : R \to R$ by $\varphi(x) = ax$. Verify that $\varphi$ is an additive group homomorphism. Prove that if $\varphi$ is surjective, then $a$ is a unit and $\varphi$ is bijective.

7. Let $X$ be a set and $a \in X$. Let $R$ be the ring of rational valued functions on $X$ with the usual pointwise operations. Prove that $\{f \in R\colon f(a) = 0\}$ is a maximal ideal of $R$.

8. Suppose $R$ is an integral domain. Show that $x, y \in R$ are associates, if and only if, they generate the same ideal in $R$.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | total (80) |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |