Midterm 3 / 2015.4.24 / MAT 4233.001 / Modern Abstract Algebra

Name: _____

Please show all work and justify your answers.

1. Let m and n be coprime natural numbers. Suppose G is an additive group with $|G| = m$. Define $\varphi\colon G \to G$ by $\varphi(x) = nx$. Prove that $\varphi \in \mathrm{Aut}(G)$.

   Hint: $nx = 0 \Rightarrow |x|$ divides n.

2. Suppose $\varphi\colon \mathbf{Z}_{14} \to \mathbf{Z}_2 \oplus \mathbf{Z}_7$ is an homorphism and $\varphi(3) = [1, 5]$. Find $\varphi(1)$.

3. Suppose $N \triangleleft G$ and $|G/N| = n$. Show that for all $x \in G$ we have $x^n \in N$.

4. Suppose $G < D_n$. Define $\varphi\colon G \to \mathbf{Z}_2$ by $\varphi(x) = 0$ if x is a rotation and $\varphi(x) = 1$ if x is a reflection. Prove that φ is a homomorphism. Explain how this proves that the rotations form a normal subgroup of D_n.

1	2	3	4	total (40)