Midterm 2 / 2015.3.27 / MAT 4233.001 / Modern Abstract Algebra

Name: ____

Please show all work and justify your answers.

1. Given natural numbers m and n, prove that in $\mathbf{Z}$ we have $\langle m\rangle \cap \langle n\rangle = \langle\, \mathrm{lcm}(m,n)\rangle$.
2. Let τ be the permutation $(2,4,5)(1,3,5,2)$. Write τ^{11} as a product of disjoint cycles.
3. Suppose $m \geq 2$, $a \in \mathbf{Z}_m$ and $\varphi\colon \mathbf{Z}_m \to \mathbf{Z}_m$ is given by $\varphi(x) = ax$.
   (a) Show that φ is an additive automorphism of $\mathbf{Z}_m$ if and only if a is a unit in $\mathbf{Z}_m$.
   (b) Show that every additive automorphism of $\mathbf{Z}_m$ is of that form with $a = \varphi(1)$.
   (c) Show that $\theta\colon \mathrm{Aut}\,\mathbf{Z}_m \to U(m)$ given by $\theta(\varphi) = \varphi(1)$ is an isomorphism.
4. In $U(20)$ find all cosets of the subgroup $\langle 11\rangle$. Explain how your result agrees with predictions of Lagrange's theorem.

1	2	3	4	total (40)