Midterm 1 / 2011.3.2 / MAT 4233.001 / Modern abstract algebra

Name: _____

Please show all work and justify your answers.

1. How many subgroups does $\mathbf{Z}_{30}$ have? $\mathbf{Z}_{32}$?

2. Are there elements of order 20 in the symmetric group S_{10}? Exhibit one or explain why there aren't any. How about elements of order 21? 23? 25? 30?

3. Suppose G is a multiplicative group, $a \in G$. Prove that $a^n = e \Leftrightarrow |a|$ divides n.

   Hint: Use the division algorithm.

4. Suppose G is a multiplicative group, $a \in G$. Prove that $a^{|G|} = e$. What conclusion can you draw about $|a|$, if $a \neq e$ and $|G|$ is prime? Explain.

   Hint: Use Lagrange's theorem.

5. Prove that $H = \{1, i, -1, -i\}$ is a multiplicative subgroup of the unit circle in the complex plane. Find two nontrivial cosets of H in the multiplicative group of all nonzero complex numbers $\mathbf{C}^*$ — one coset that is a subset of the unit circle and one not. Sketch H and the two cosets you found. How many distinct cosets does H have in $\mathbf{C}^*$?

   Hint: Pick two points in $\mathbf{C}^*$ not in H — one on the unit circle $\{a + ib\colon a^2 + b^2 = 1\}$ and one not.

1	2	3	4	5	total (50)	%

Prelim. course grade: %