Midterm 1 / 2010.3.3 / MAT 4233.001 / Modern abstract algebra

Name: _____

Please show all work and justify your answers.

1. Suppose $a \in \mathbf{Z}_n$. Prove $a \in U(n)$ if and only if a is relatively prime to n. What is $|U(n)|$ if n is prime? Explain. What is the multiplicative inverse of 5 in $\mathbf{Z}_{18}$?

2. Prove or disprove $U(8) \cong U(12)$.

3. Let $H = \{(), (12)(34), (13)(24), (14)(23)\}$. Prove that H is a subgroup of A_4 (you may use the word *similarly* as appropriate). List all the cosets of H in A_4. Is H isomorphic to $\mathbf{Z}_4$? Explain.

4. Suppose G is a group with $|G| = 11$. Prove or disprove that G must be cyclic.

5. Suppose G is a group with $|G|$ a positive integer power of a prime p. Prove that G has an element of order p.

1	2	3	4	5	total (50)	%

Prelim. course grade: %