Name: _____

Please show all work and justify your answers.

1. Suppose $a \in \mathbf{Z}_n$. Prove $a \in U(n)$ if and only if a is relatively prime to n. What is $|U(n)|$ if n is prime? Explain. What is the multiplicative inverse of 7 in $\mathbf{Z}_{24}$?
2. Let G be the multiplicative subgroup of $\mathbf{R}^*$ generated by 1. Prove or disprove $G \cong \mathbf{Z}_2$.
3. Let $S = \{z \in \mathbf{C}\colon |z| = 1\}$ be the unit circle in the complex plane. Prove that S is a multiplicative subgroup of $\mathbf{C}^*$. Sketch S and two nontrivial cosets of S in $\mathbf{C}^*$.
4. Suppose G is an abelian group with $|G| = 12$. Must G be cyclic? Explain.
5. Suppose G is a group with $|G| = 27$. Prove that G has an element of order 3.
6. Prove that $\{\sigma \in S_4\colon \sigma(4) = 4\}$ is a subgroup of S_4. Is it abelian? Is it a normal subgroup of S_4? Prove your assertions.
7. Find the quotient and remainder of $x^4 + 2x^3 + 2x^2 - x + 1$ divided by $2x^2 + 1$ in $\mathbf{Z}_3[x]$.
8. Let A be the ideal generated by $x - 1$ in $\mathbf{Q}[x]$. Is A a prime ideal? Maximal? Explain.

1	2	3	4	5	6	7	8	total (80)