Midterm 1 / 2009.3.5 / MAT 4233.001 / Modern abstract algebra

Name: _____

Please show all work and justify your answers.

1. Prove that $a \in \mathbf{Z}_n$ has a multiplicative inverse if and only if a is relatively prime to n. What is the multiplicative inverse of 3 in $\mathbf{Z}_{10}$?

2. Suppose G is a group where each nontrivial element has order 2. Prove that G is abelian.

3. Suppose G is a cyclic group of order 18. How many subgroups does it have? Explain.

4. Suppose G is a group with $|G|$ a positive integer power of 2. Prove that G has an element of order 2.

5. Let $H = \{(), (12)(34), (13)(24), (14)(23)\}$. Prove that H is a subgroup of S_4 (you may use the word *similarly* as appropriate). What is its index? Is H isomorphic to $\mathbf{Z}_4$? Explain.

1	2	3	4	5	total (50)	%

Prelim. course grade: %