Final exam / 2008.5.9 / Modern abstract algebra / MAT 4233.001

Name: ______________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Suppose G is a group such that $\forall a, b, c \in G\ ab = ca \Rightarrow b = c$. Prove that G is abelian.
2. Show that in a finite group the number of all elements of order 3 is even.
3. Let $G = GL(n, \mathbf{Q})$ be the multiplicative group of invertible $n \times n$ matrices with rational coefficients and $H = SL(n, \mathbf{Q}) = \{A \in G\colon \det A = 1\}$. Prove that H is a subgroup of G. Prove or disprove that H is normal in G.
4. Let G and H be as in the preceding problem. Suppose $A, B \in G$ and $\det A = \det B$. Prove that A and B belong to the same left coset of H.
5. Prove that for $n \geq 3$ the symmetric group S_n has trivial center. What is $Z(S_2)$?
6. Let A be the set of all elements of the ring $\mathbf{Z} \oplus \mathbf{Z}$ whose first coordinate is even. Prove that A is an ideal. Is it maximal? Prove your assertion.
7. Suppose $\varphi : R \to S$ is a ring homomorphism from a ring with unity R to an integral domain S such that $\varphi(R) \neq \{0\}$. Prove that $\varphi(1) = 1$.
8. Prove that $x^p + x + 1$ and $2x + 1$ determine the same function $\mathbf{Z}_p \to \mathbf{Z}_p$.

1	2	3	4	5	6	7	8	total (80)