University of Texas at San Antonio

Real Analysis II, MAT 4223
Exam $\mathcal{N}^{\underline{o}}2$, 4/8/92
Instructor: D. Gokhman

Name: ______________________________
Make sure to include statements of theorems used in your proofs.

1. (20 pts.) Test the following series for convergence:

$$\text{(a)} \sum_{n=2}^{\infty} \frac{1}{n\log(n)} \qquad \text{(b)} \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

2. (20 pts.) Find the interval of convergence for the following series:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(x+1)^n}{n} \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^3}$$

3. (30 pts.) Find the pointwise limit of the following sequences as $n \to +\infty$ for $x \in [0, +\infty)$ and in each case determine (with proof) whether the convergence is uniform in $[0, +\infty)$:

$$\text{(a)} \frac{x}{1+nx} \qquad \text{(b)} \frac{nx^2}{1+nx}$$

4. (30 pts.) Let

$$f(x) = \sum_{i=1}^{\infty} \frac{\cos(nx)}{(n-1)!}.$$

   (a) Prove that the above series converges for each $x \in \mathbf{R}$.

   (b) Prove that $f(x)$, thus defined, is Riemann integrable in any interval $[a, b]$.

   (c) Evaluate

$$\int_0^{\frac{\pi}{2}} f(x)\,dx.$$