Midterm 2 / 2016.11.30 / MAT 4213.001 / Real Analysis I

Name: _____

Please show all work. If you use a theorem, name it or state it.

1. Suppose $f: \mathbf{R} \to \mathbf{R}$ is increasing and $c \in \mathbf{R}$. Prove that f has a left limit at c.
2. Suppose $f: \mathbf{R} \to \mathbf{R}$ is differentiable and $c \in \mathbf{R}$. If $\lim_{x \to c} f'(x) = L$, show that $f'(c) = L$.
3. Suppose $f(x) = x^2 \sin(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$.
   (a) For each $x \in \mathbf{R}$ find $f'(x)$.
   (b) Show that f' is not continuous at 0.
4. Find the limits at 0 and ∞ of $\frac{1}{x} \sin(x)$ and prove your results.
5. Let $f(x) = \cos(x)$.
   (a) What is the n-th Taylor polynomial for $f(x)$?
   (b) Show that for any $x \in \mathbf{R}$ the n-th remainder converges to 0 as $n \to \infty$.

   Hint: $R_n(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!} x^{n+1}$ for some c between 0 and x.

1	2	3	4	5	total (50)