Real Analysis I / MAT 4213.001

Midterm 1 / October 7, 1998 / Instructor: D. Gokhman

Name: ______________________________

Throughout, suppose (x_n) and (y_n) are sequences of real numbers, $x, y \in [-\infty, \infty]$, x is a partial limit of (x_n) and y is a partial limit of (y_n).

1. (30 pts.) Determine whether each of the following statements is true in general. If true, prove it. If false, give a specific counterexample.

   (a) If $\exists m \ \forall n \geq m \ x_n \geq 0$, then $x \geq 0$.

   (b) If $\forall m \ \exists n \geq m \ x_n \geq 0$, then $x \geq 0$.

   (c) If $\exists m \ \forall n \geq m \ y_n > 0$, then $y > 0$.

   (d) If 0 is a partial limit of $x_n - y_n$, then x is a partial limit of (y_n).

   (e) If $x_n - y_n \to 0$, then x is a partial limit of (y_n).

   (f) $x + y$ is a partial limit of the sequence $(x_n + y_n)$.

2. (10 pts.) Suppose $A \subseteq \mathbf{R}$ and a is a limit point of A. Prove that there exists a sequence in $A \setminus \{a\}$ that converges to a.

3. (10 pts.) Let $a = \liminf x_n$ and $b = \limsup x_n$. Suppose U is an open interval containing the closed interval $[a, b]$. Prove that $\exists m \ \forall n \geq m \ x_n \in U$.

1(a-c)	1(d-f)	2	3	total (50)	%