University of Texas at San Antonio

Real Analysis I, MAT 4213
Exam $\mathcal{N}^{\underline{o}}$1, 10/12/92
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Name: ______________________________

1. (40 pts.) Suppose (X, d) is a metric space.
   Given $x \in X$ and a subset $S \subseteq X$ define
   $$d^*(x, S) = \inf_{s \in S} d(x, S).$$
   Prove that
   (a) $d^*(x, S) = 0 \Leftrightarrow x \in \overline{S}$.
   (b) If S is closed and $x \notin S$, then $d^*(x, S) > 0$.
   (c) S is dense (i.e. $\overline{S} = X$) $\Leftrightarrow \forall x \in X \; d^*(x, S) = 0$.
   (d) S is dense $\Leftrightarrow$ ($\forall$ open nonempty $U \subseteq X$) $S \cap U \neq \emptyset$.

2. (40 pts.) True of false questions, circle your choice. If you choose TRUE, prove the assertion somewhere in the lower part of the page. If FALSE, provide a counterexample.

T F (a) A finite subset of $\mathbf{R}$ is compact.

T F (b) A countable subset of $\mathbf{R} \setminus \mathbf{Q}$ cannot be dense in $\mathbf{R}$.

T F (c) If $E \subseteq \mathbf{R}$, then $\mathring{E} = \mathring{\overline{E}}$ or $\overline{E} = \overline{\mathring{E}}$.

T F (d) $\mathbf{R}$ does not have an open cover with a finite subcover.

3. (20 pts.) Classify all nonempty connected $E \subseteq \mathbf{R}$ that are
   (a) compact. (b) finite.