University of Texas at San Antonio

Real Analysis I, MAT 4213
Exam $\mathcal{N}^{\underline{o}}2$, 11/21/91
Instructor: D. Gokhman

Name: _____

1. (20 pts.) Suppose $f, g : \mathbf{R} \to \mathbf{R}$ are continuous functions. Show that the set $\{x : f(x) = g(x)\}$ is closed in $\mathbf{R}$.

2. (20 pts.) Find all cluster points of the sequence $\{a_n\}$, where $a_n = \cos(n\pi) + \cos(\dfrac{1}{n\pi})$.

3. (20 pts.) Classify all functions $f : \mathbf{R} \to \mathbf{R}$ which are continuous and such that $f(\mathbf{R}) \subsetneq \mathbf{Q}$. Justify your answer.

4. (20 pts.) In each of the following determine whether the limit exists. If so, find the limit and prove your assertion.

   (a) $\displaystyle\lim_{x \to 0} \frac{1 - \cos(x)}{x}$,

   (b) $\displaystyle\lim_{x \to 0} x \sin(\frac{1}{x})$.

5. (20 pts.) Suppose $f : \mathbf{R} \to \mathbf{R}$.
   Let $D = \left\{ \dfrac{f(y) - f(x)}{y - x} : x, y \in \mathbf{R}, x \neq y \right\}$. Show that if D is a bounded set, then f is uniformly continuous.