Midterm 3 / 2016.4.29 / MAT 3623.001 / Differential Equations II

Name: ______________________________

Please show all work.

1. Let $f(t) = 2 - |t|$. Obtain the first 3 nonzero terms of the Fourier expansion for f on the interval $[-2, 2]$. On the same graph sketch the function and the three partial sum approximations.

2. Solve the vibrating string equation $u_{tt} = c^2 u_{xx}$ for a string of length L with initial conditions $u(x, 0) = \sin \frac{3\pi x}{L}$, $u_t(x, 0) = 0$. On the same graph sketch $u(x, t)$ as functions of x for three different fixed values of t (starting with $t = 0$) to illustrate time evolution of the solution.

3. Find the steady state temperature of the disc $r \leq 5$, if the boundary $r = 5$ is held at $u(5, \theta) = 23 - 2\sin(3\theta)$ (in polar coordinates).

   Hint: For the Cauchy-Euler equation look for solutions in the form of powers.

Fourier series: $f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L}]$,

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(t)\, dt, \quad a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L}\, dt \ (n \geq 1), \quad b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L}\, dt$$

Laplacian: $\nabla^2 u = u_{xx} + u_{yy} = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}$

1	2	3	total (30)