Midterm 2 / 2016.3.25 / MAT 3623.001 / Differential Equations II

Name: _____

Please show all work.

1. Consider the dynamical system $x'(t) = -7x(t) + 3y(t), y'(t) = -18x(t) + 8y(t)$.
   (a) Show that the origin is the unique equilibrium of the system and determine its stability.
   (b) Find the invariant manifolds.
   (c) Sketch the invariant manifolds and a few trajectories of the system.
2. Consider the boundary value problem $x''(t) = t^2, x(0) = x(2) = 0$
   (a) Solve this problem using the variation of parameters formula
   $$x(t) = x_1(t) \int_t^2 \frac{f(s)x_2(s)\,ds}{W(s)} + x_2(t) \int_0^t \frac{f(s)x_1(s)\,ds}{W(s)},$$
   where f is the right-hand side, x_1, x_2 are linearly independent solutions of the homogeneous equation satisfying boundary conditions (one each) and W is their Wronskian.
   (b) What is the Green's function $G(t, s)$ for this boundary value problem?
   (c) Graph $G(1, s)$ on the interval $0 \le s \le 2$.
3. Solve the boundary value problem $x''(t) - x(t) = t, x(0) + x'(0) = 1, x(1) - x'(1) = 1$.

1	2	3	total (30)