

Name: _____

Please show all work.

1. Solve the Clairaut equation $x = tx' - (x')^2$ and find the singular solution. Sketch the singular solution and several line solutions.
2. Let $A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$. Show that the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ leads to a periodic solution. Sketch.
3. Find the fundamental solution to the Airy equation $x'' = tx$ in power series form. Find the first 3 nonzero terms of each power series. Based on the form of the equation alone, what is your prediction for the radius of convergence of the power series?
4. For each of the following linear differential equations find and classify all singular points. For each regular singular point find and solve the indicial equation.
 - (a) $(t^2 - t)x'' + tx' + 3x = 0$
 - (b) $(t^2 - 1)^2x'' - (t - 1)x' + 7x = 0$

1	2	3	4	total (40)