University of Texas at San Antonio

Independent Study, MAT 4913
Final Exam, 5/11/92
Instructor: D. Gokhman

Name: ______________________________

1. (20 pts.) Find the radius of convergence of the following power series:
   (a) The Maclaurin series for $\left(x^2 - 2x + 2\right)^{-1}$.
   (b) $\sum_{k=0}^{\infty} a_{n+1}(n+1)x^n$
   (assume the radius of convergence of $\sum_{k=0}^{\infty} a_n x^n$ is ρ).
2. (20 pts.) Find all singular points of the following equations. Determine whether they are regular or irregular. If a singular point is regular, derive and solve the indicial equation.
   (a) $x^3 y'' + \alpha x y' + \beta y = 0$, where $\alpha \neq 0$
   (b) $x(1-x)y'' + [\gamma - (1+\alpha+\beta)x]y' - \alpha\beta y = 0$.
3. (20 pts.) Find the general series solution to $y'' = xy$.
4. (20 pts.) Use second order Taylor series with remainder to derive the local formula error for Euler's method with step h for the equation $y' = F(x, y)$.
5. (20 pts.) Let $U \subseteq \mathbf{R}^2$ be an open set containing the origin and assume that $f(x, y)$ keeps constant sign in U. Consider the following equation
$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f & 1 \\ -1 & f \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$
   (a) Show that the origin is a critical point of this equation.
   (b) Depending on the sign of f determine the stability of the origin by constructing an appropriate Liapunov function.