

**University of Texas at San Antonio**

Engineering Analysis II, MAT 3263

Exam N<sup>o</sup>2, 11/21/91

Instructor: D. Gokhman

Name: \_\_\_\_\_

1. (10 pts.) Classify all continuous functions  $f : \mathbf{R} \rightarrow \mathbf{R}$  satisfying the following property: if  $T > 0$ , then  $f$  is periodic with period  $T$ .
2. (10 pts.) Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be a periodic function of period 1 defined by  $f(x) = \ln x$  for  $x \in (1, 2]$ . Sketch  $f$ . Let  $g = \sum_{n=-\infty}^{\infty} c_n e^{i2n\pi x}$ , where  $c_n = \int_1^2 \ln(x) e^{-i2n\pi x} dx$ . Determine the function  $f(x) - g(x)$ .
3. (30 pts.) Let  $\omega > 0$ . For  $\eta \neq \pm \frac{\omega}{2}$  find the steady state solution of the differential equation  $y'' + \omega^2 y = \cos^2(\eta t)$ .
4. (20 pts.) Let  $\hat{f}(\omega)$  denote the Fourier transform of  $f(x)$ . Let  $a > 0$  and  $g(x) = f(ax)$ . Express  $\hat{g}$  in terms of  $\hat{f}$ .
5. (30 pts.) Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be a periodic function of period 2 defined by  $f(x) = e^{|x|}$  for  $x \in (-1, 1]$ . Sketch  $f$ . Find the Fourier series expansion of  $f$ .