University of Texas at San Antonio

Engineering Analysis II, MAT 3263

Final Exam, 12/10/91

Instructor: D. Gokhman

Name: ______________________________

1. (25 pts.) Suppose A, B, C are points in $\mathbf{R}^3$. Assume that the corresponding position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ form a linearly independent set.

   (a) Construct a basis for the vector subspace of $\mathbf{R}^3$ that is a plane parallel to ABC.

   (b) Show that $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$ is perpendicular to the plane ABC.

2. (25 pts.) Let $\mathbf{F}(x, y, z) = y \sin z \mathbf{j} + z \cos y \mathbf{k}$.

   (a) Calculate the Jacobian matrix of $\mathbf{F}$.

   (b) Calculate the trace (sum of the diagonal elements) of this matrix. What is the more familiar name for this operator?

   (c) Calculate the gradient of this last expression. What is the more familiar name for this operator?

   (d) Calculate the curl of $\mathbf{F}$.

3. (25 pts.) Let $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + xyz^2\mathbf{k}$.

   (a) Sketch the circle $x^2 - 2x + y^2 = 2$.

   (b) Calculate directly $\int \mathbf{F} \cdot d\mathbf{r}$ counterclockwise once around the circle.

   (c) Use the theorem of Stokes to check your answer.

4. (25 pts.) Let $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$.

   (a) Sketch the surface $x^2 + y^2 = z^2$, $1 \leq z \leq 2$.

   (b) Calculate $\int\int \mathbf{F} \cdot \mathbf{n} \, dS$ over this surface.

   Hint: $\cos^3\theta$ and $\sin^3\theta$ can be integrated graphically.

5. (25 pts.) Suppose $\mathcal{S}$ is a sphere in $\mathbf{R}^3$ and $f(x, y, z)$ is a harmonic function (i.e. $\nabla^2 f = 0$). Let $\frac{\partial f}{\partial n}$ denote the directional derivative of f along the normal unit vector $\mathbf{n}$ to the surface $\mathcal{S}$. Show that $\int\int_{\mathcal{S}} \frac{\partial f}{\partial n} \, dS = 0$.

6. (25 pts.) Suppose $f(t)$ is periodic with period $2L$,

$$f(t) = \begin{cases} A\sin\left(\frac{\pi}{L}t\right) & \text{for } 0 \le t \le L, \\ 0 & \text{for } -L < t < 0. \end{cases}$$

(a) Sketch f over three periods.

(b) Find the Fourier series expansion of f.

7. (25 pts.) Let $f(x) = \begin{cases} e^{-x} & \text{for } x \ge 0, \\ 0 & \text{for } x < 0. \end{cases}$

(a) Sketch f.

(b) Find $\hat{f}$ (the forward complex Fourier transform of f).

(c) Sketch $\hat{f}$.

8. (25 pts.) Find the general solution to $x^2 u_{xy} + 3y^2 u = 0$.