

Name: _____

1. (40) Find the general solution of the homogeneous differential equation

$$tx^2 dx + (t^3 - x^3) dt = 0.$$

2. (40) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = \ln(t) e^{-t}.$$

3. (40) Find the general solution of the system

$$\frac{dx}{dt} - x + 2\frac{dy}{dt} = e^t, \quad \frac{dx}{dt} - 5x + \frac{dy}{dt} = -2e^t.$$

4. (40) Express the following function in terms of unit step functions and calculate its Laplace transform.

$$f(t) = \begin{cases} 1, & t < 1 \\ 0, & 1 \leq t < 2 \\ (t-2)^2, & 2 \leq t \end{cases}$$

Extra credit (10): What happens if $(t-2)^2$ is replaced by t ?

5. (40) Find the solution of the initial value problem

$$\frac{dx}{dt} + 5x = \delta(t-2), \quad x(0) = 10.$$

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- $\mathcal{L}\{t^n\} = n!/s^{n+1}$ $\mathcal{L}\{e^{at}f\} = \mathcal{L}\{f\}(s-a)$
 $\mathcal{L}\{\delta(t)\} = 1$ $\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{f\}$
 $\mathcal{L}\{f'\} = s\mathcal{L}\{f\} - f(0)$

1	2	3	4	5	total (200)