Final exam / 2009.12.11 / Calculus for Applications / MAT 3243.001

Name: ______________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. In each case determine whether the limit exists, and if so, find the limit.

$$\text{(a)} \lim_{[x,y]\to 0} \frac{x^2y+y^3}{\sqrt{x^2+y^2}} \qquad \text{(b)} \lim_{[x,y]\to 0} \frac{x^3y+y^4}{x^4+y^4}$$

2. The temperature distribution (in degrees Fahrenheit) at position $[x, y]$ (in miles) is given by $T(x, y) = 98 - x^3y^2$. You start walking northwest from $[-1, 1]$ at 3 miles per hour. How fast is the temperature changing?

3. Let $f = \cos(1+x^2+y)$. Compute the Hessian matrix for $f$ and find the quadratic Taylor approximation to $f$ at the origin.

4. A Petri dish 3 inches in diameter is used to grow a culture of H1N1 and the population density is given by $d(x, y) = x^2 + 2y^2 - y + 2$ in billions of virii per square inch. Where is the population density the lowest? The highest?

5. Sketch the solid enclosed by the surfaces $z = 3 - \sqrt{x^2+y^2}$ and $z = 0$. Use triple integration in cylindrical coordinates to compute its volume.

6. Find the scalar potential for the vector field $F = [z\cos x, 2y, \sin x]$ or show that such a potential doesn't exist.

7. Integrate $\omega = 2x\,dy - 3y\,dx$ around the circle of radius 3 centered at the origin counterclockwise. Compute the same integral using Green's theorem.

8. Compute the flux of $F = [x, y, z]$ through the surface $z = 1 - x^2 - y^2, z \geq 0$ oriented with the upward normal both directly and also using the divergence theorem.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | total (80) |
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