Final exam / 2008.12.12 / Calculus for Applications / MAT 3243.001

Name: ____

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Sketch and label 5 level sets of $f(x, y) = x^2 - y^3$, including one at level 0.

2. In each case determine whether the limit exists, and if so, find the limit.

$$\text{(a)} \lim_{[x,y]\to 0} \frac{xy}{x^2 + y^2} \qquad \text{(b)} \lim_{[x,y]\to 0} \frac{x^3 - y^3}{x^2 + xy + y^2}$$

3. If a trilobyte crawls south at 2 cm/s, it notices an increase in temperature at the rate of 1°/s. If it crawls west at 1 cm/s, the temperature increases by 3°/s. What is the rate of change of temperature if the cucaracha crawls southeast at 3 cm/s?

4. Find the divergence and curl of $[x^2 y, \cos(xyz), y^2 z]$.

5. Let $f = e^{x+y^2}$. Compute the Hessian matrix for f and find the quadratic Taylor approximation to f at the origin.

6. A solid is bounded by the coordinate planes and the plane $x + 2y + 7z = 14$. Set up, but do not evaluate the iterated integral for the volume with the order of integration y, x, z.

7. Integrate $\omega = x\,dx + y\,dy$ along the straight line segment from $[-1, -1]$ to $[1, 1]$. Had we chosen a different path from $[-1, -1]$ to $[1, 1]$, would the integral remain the same? Explain.

8. Find first a parametric formula and then an equation for the plane in $\mathbf{R}^3$ tangent to the surface $[s + t, st, \sin(st)]$ at $[1, 0, 0]$.

9. Parametrize the paraboloid $z = 2 - x^2 - y^2, z \geq 1$ oriented with the upward normal. Compute the flux of $\mathbf{F} = [x, y, -2z]$ through this surface. Would the flux of $\mathbf{F}$ through the unit disc in the $z = 1$ plane differ? Explain.

1	2	3	4	5	6	7	8	9	total (90)