Final exam / 2004.12.13 / Calculus for Applications / MAT 3243

Name: ______________________________

Please show all work and justify your statements. Label sketches, draw conclusions using complete sentences including units, and box your final answers as appropriate.

1. Find a parametric formula for the line tangent to the path $[e^{t^2-4}, 2t, \ln(t-1)]$ at $(1, 4, 0)$.

2. A surface in $\mathbf{R}^3$ is given by $\ln(xy) = \sin(yz)$. Find an equation for the plane tangent to this surface at $(1, 1, 0)$.

3. Let $f(x, y, z) = x^2yz^3$ and $F(x, y, z) = [0, \ln(f(x, y, z)), 0]$.
   (a) Compute the directional derivative of f along the direction given by $(1, 0, 1)$.
   (b) Compute the curl and the divergence of the vector field $F + \nabla f$.

4. Suppose $z = f(u, v)$, where $u = x + y$ and $v = xy$. Express the partial derivatives of z with respect to x and y in terms of the partial derivatives of f with respect to u and v.

5. Integrate $z\,dx + x\,dy$ along the straight line segment from $(1, 1, 2)$ to $(2, 5, 3)$. Had we chosen another path between these points, would the integral remain the same? Explain.

6. Find all ω on $\mathbf{R}^3$ such that $d\omega = 3y^2\,dx + (6xy + z^2)\,dy + 2yz\,dz$.

7. Compute the flux of $F = [x, y, 2z^2]$ through the cylinder $x^2 + z^2 = 4$, $0 \le y \le 3$ oriented with the outward normal vector.

8. Given a closed surface oriented with the outward normal, how is the integral of $z\,dy\,dx$ through the surface related to the volume of the region enclosed by the surface? Explain.

9. ACME roadrunner traps are made of wood and steel, each costing p and q dollars per unit respectively. The number of traps ACME can produce using x units of wood and y units of steel is cx^ay^b, where a, b, and c are positive constants. If ACME's budget for raw materials is B dollars, what is the largest number of traps they can produce?

10. A quonset hut is shaped like a half cylinder of radius 5 m and length 40 m. The hut is filled with hay, which is compressed under its own weight in such a way that the density varies linearly from 100 kg/m^3 at the top to 200 kg/m^3 at the bottom. Set up, but do not evaluate, an iterated integral for the total mass of hay in the hut. Sketch the hut and indicate your coordinate system in the sketch.

1	2	3	4	5	6	7	8	9	10	total (100)