Final Exam / 2003.12.12 / Calculus for Applications / MAT 3243

Name: ____________________

Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box your final answers as appropriate.

1. A surface in $\mathbf{R}^3$ is given by $xz + \cos(yz) = 2$. Find an equation for the plane tangent to this surface at $(1, 0, 1)$.

2. Find parametric formulas for

   (a) The line tangent to the path $[\cos(\pi t), 3^t - 1]$ at $[1, 8]$.

   (b) The plane tangent to the surface $[st, s + t, e^{st}]$ at $[0, 1, 1]$.

3. (a) Compute the curl and the divergence of the vector field $[xyz, 1, \ln(x^2 + y^2 + z^2)]$.

   (b) Let $\omega = xy$ and $\eta = y\,dx + x\,dz$. Find and simplify $d\omega \wedge \eta$ and $d\omega \wedge d\eta$.

4. In calm seas an oil tanker hits an underwater sand bar and springs a leak. After a while, the radius of the resulting oil slick is 5 km and is increasing at the rate of 100 m/h. At the same time the thickness of the slick is 1 mm and is increasing at the rate of 0.02 mm/h. Estimate the rate at which the oil leaks out of the tanker.

5. Compute the work done by the force $\mathbf{F} = [y, 0]$ in moving a particle along the top half of the unit circle in $\mathbf{R}^2$.

6. What is the integral of $\omega = x\,dx + y\,dy + z\,dz$ around any closed loop in $\mathbf{R}^3$? Explain.

7. Find a potential for the vector field $\mathbf{F} = [3x^2 - 5y, 9y^2 - 5x]$.

8. Find the flux of $\mathbf{F} = [3x, 5y, z]$ through the surface $x^2 + y^2 = 9, -2 \le z \le 3$.

9. Let M be the unit disc in $\mathbf{R}^2$ and $\omega = -y\,dx + x\,dy$. Verify the fundamental theorem of calculus $\int_{\partial M} \omega = \int_M d\omega$ in this case.

10. A fuel rod has the shape of a solid cylinder $r \le 1$, $0 \le z \le 10$. If the density of fuel is $\rho(r, \theta, z) = (4 - r^2)(12 - z)$, what is the total mass of the rod?

1	2	3	4	5	6	7	8	9	10	total (100)