

Name: _____

Please show all work and explain your answers. Sketch.

1. (20 pts.) The position (in km) of a cruise missile is given as a function of time from launch (in minutes) by $x(t) = t$, $y(t) = t^2$, $z(t) = 3t - t^2$.

(a) When does the missile hit its ground target? What are the target's coordinates? How far is the target from the launch site?

(b) What are the missile's velocity and speed upon impact?

2. (20 pts.) Let f be the transformation of the plane given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2x + y^2 \\ x^2y \end{bmatrix}$.

Find the linear approximation to f at the point $p = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

3. (20 pts.) Suppose $f(x, y)$ is a differentiable function from the plane to the reals, and we have new coordinates $s = 3x + y$ and $t = x - y$.

(a) Express the first partial derivatives of f with respect to x and y in terms of those with respect to s and t .

(b) Use the formulas derived in part (a) to express the mixed partial derivative $\frac{\partial^2 f}{\partial x \partial y}$ in terms of the coordinates s and t .

4. (20 pts.) Compute the following volumes

(a) volume under the graph of $z = \begin{cases} x^2 \log y & \text{for } (x, y) \text{ in } D = [0, 1] \times [1, 2] \\ 0 & \text{for } (x, y) \text{ outside } D \end{cases}$

(b) volume of a regular pyramid with height 60 m and a square base of side 100 m

1	2	3	4	total (80)	(%)