Name: _____

Please show all work and explain your answers. Sketch.

1. (20 pts.) The position (in km) of a cruise missile is given as a function of time from launch (in minutes) by $x(t) = 2t$, $y(t) = t^3$, $z(t) = 4t - t^2$.
   (a) When does the missile hit its ground target? What are the target's coordinates? How far is the target from the launch site?
   (b) What are the missile's velocity and speed upon impact?

2. (20 pts.) Let f be the transformation of the plane given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 3x - y^2 \\ x^3 y \end{bmatrix}$.
   Find the linear approximation to f at the point $p = (2, -3)$.

3. (20 pts.) Suppose $f(x, y)$ is a differentiable function from the plane to the reals, and we have new coordinates $s = 2x - y$ and $t = 3y - x$.
   (a) Express the first partial derivatives of f with respect to x and y in terms of those with respect to s and t.
   (b) Use the formulas derived in part (a) to express the second partial derivative of f with respect to y in terms of the coordinates s and t.

4. (20 pts.) Use Cavalieri's principle to compute the volume of a regular pyramid with height 50 m and a square base of side 80 m.

5. (20 pts.) Find the arc length of the helix $\gamma(t) = (5\sin(t), 5\cos(t), -2t)$ between $(0, -5, 2\pi)$ and $(0, -5, -2\pi)$. Sketch.

6. (20 pts.) Find the flux of $F(x, y, z) = (2, x, z)$ through the surface $x^2 + y^2 + z^2 = 9, z \leq 0$. Sketch the surface and F at several points on the surface.

7. (20 pts.) Find the work done by the force field $F(x, y, z) = (x + yz^2, xz^2, 2xyz)$ in moving a particle from $(-1, 1, 2)$ to $(1, -2, -1)$. Does it matter along which path the particle is moved? Explain.

8. (20 pts.) Let $F = (6xz^2, 2y^3, 6zx^2)$ and $\omega = F \cdot dS$, where $dS = (dy\,dz, dz\,dx, dx\,dy)$.
   (a) Compute $d\omega$.
   (b) Use the general fundamental theorem of calculus to express the flux of F through the unit sphere as a density integral with respect to $dx\,dy\,dz$. Evaluate this integral.

1	2	3	4	5	6	7	8	total (160)	(%)