Calculus for Applications / MAT 3243
Midterm 1 / October 20, 1999 / Instructor: D. Gokhman

Name: _____

Please show all work and box the answers, where appropriate.

1. (10 pts.) Find parametric formulas for the following geometric objects. Sketch.
   (a) Line in $\mathbf{R}^3$ through $(0,0,1)$ and $(1,1,0)$.
   (b) Plane (through the origin) containing the vectors $(1,1,0)$ and $(1,0,1)$.
2. (5 pts.) Convert $(-1,\sqrt{3},2)$ to cylindrical coordinates.
3. (10 pts.) Let $f\colon \mathbf{R}^2 \to \mathbf{R}^2$ be reflection with respect to the line $y=-x$.
   (a) Find $f(\widehat{\imath})$ and $f(\widehat{\jmath})$.
   (b) What matrix represents f with respect to the standard basis?
   (c) Given $v=(x,y)$, find $f(v)$ in terms of x and y.
4. (10 pts.) Sketch three level curves in $\mathbf{R}^2$ and then the graph of $z^2 = x^2 - y^2$.
5. (10 pts.) Evaluate each of the following limits or explain why the limit fails to exist.

   (i) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\tan(x^2+y^2)}$ (ii) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$

6. (10 pts.) Let $f(x,y) = (x+3)^2 + (y+1)^2$, $g(x,y) = 10 + mx + ny$, and $\varepsilon(x,y) = f(x,y) - g(x,y)$.
   (a) Find m and n such that g is the tangent plane to f at $(0,0,10)$.
   (b) With these values of m and n compute $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\varepsilon(x,y)}{\sqrt{x^2+y^2}}$.

1	2	3	4	5	6	total (55)	%