Calculus for Applications / MAT 3243
Final / December 14, 1999 / Instructor: D. Gokhman

Name: _____ Pseudonym: _____

Please show all work and box the answers, where appropriate.

1. (10 pts.) Sketch the given points and convert between the specified coordinate systems:
   (a) $(-1,-1,-1)$ from cartesian to cylindrical.
   (b) $(\rho,\varphi,\theta) = (2, 5\pi/6, 3\pi/2)$ from spherical to cartesian.

2. (10 pts.) Let $f\colon \mathbf{R}^2 \to \mathbf{R}^2$ be projection to the line $x + 2y = 0$. Find the matrix that represents f with respect to the standard basis and a general formula for $f(x,y)$.

3. (10 pts.) Sketch at least three level curves in $\mathbf{R}^2$ and then the graph in $\mathbf{R}^3$ of $z = y^2 - x^3$.

4. (10 pts.) Let $f(x,y) = x + 2y^2$, $g(x,y) = b + mx + ny$, and $\varepsilon(x,y) = f(x,y) - g(x,y)$.
   (a) Find b, m, and n such that g is the tangent plane to f at $(2,1,4)$.
   (b) With these values of b, m, and n show that $\varepsilon(x,y)/d(x,y) \to 0$ as $(x,y) \to (2,1)$, where $d(x,y)$ is the distance from (x,y) to $(2,1)$.
   (Hint: change variables: let $h = x - 2$, $k = y - 1$.)

5. (10 pts.) Let $g(x,y,z) = xyz$ and $f(u) = \cos(u)\,\widehat{j} + \ln\left(u^2+1\right)\widehat{k}$.
   Find $f \circ g$, $D(f)$, $D(g)$, and $D(f \circ g)$.

6. (10 pts.) Find an equation for the plane tangent to $x^3 - y^2 z = 7$ at $(2,1,1)$.

7. (10 pts.) Consider a curved segment $s(t) = (t, t^{3/2}, t)$, $0 \le t \le 1$.
   (a) Find a parametric formula for the line tangent to the curve at $(0,0,0)$. Sketch.
   (b) Find the arclength of the above curved segment.

8. (10 pts.) Integrate $y\,dx - y^2 x\,dy$ along the straight line segment from $\widehat{j} - \widehat{i}$ to $\widehat{i} - \widehat{j}$.

9. (10 pts.) Let $F(x,y,z) = (yz, y + zx, xy + z^2)$. Find a function $f(x,y,z)$ such that $D(f) = F$ and use it to integrate $F \cdot ds$ along an arbitrary path from the origin to $(-1,-1,-1)$.

1	2	3	4	5	6	7	8	9	total (90)