Calculus for Applications / MAT 3243.001

Midterm 1 / October 12, 1998 / Instructor: D. Gokhman

Name: _____

Please show all work and box the answers.

1. (20 pts.) Let P be the plane in $\mathbf{R}^3$ spanned by $\widehat{\imath}-3\widehat{k}$ and $\widehat{\jmath}+2\widehat{k}$. Let $p=\widehat{\imath}-\widehat{\jmath}+2\widehat{k}$. Let L be the line through 0 and p. Let Q be the plane containing p parallel to P.
   (a) Express Q and L in parametric form.
   (b) Express Q as the locus of a linear equation.
   (c) Is L perpendicular to P? Explain.

2. (21 pts.) Sketch the following manifolds and express them in parametric form:
   (a) Straight line in $\mathbf{R}^3$ through $\widehat{k}$ in the direction $\widehat{\imath}+\widehat{\jmath}$.
   (b) The ray (half-line) in $\mathbf{R}^3$ from 0 in the direction $-\widehat{\imath}-\widehat{k}$.
   (c) Straight line segment in $\mathbf{R}^2$ from $\widehat{\imath}$ to $\widehat{\jmath}$.
   (d) The circle in $\mathbf{R}^2$ of radius 2 centered at 0.
   (e) The circle in $\mathbf{R}^2$ of radius 2 centered at $\widehat{\imath}+3\widehat{\jmath}$.
   (f) Right half of the circle in (d).
   (g) Circle in $\mathbf{R}^3$ of radius 3 centered at $\widehat{\jmath}$ parallel to the x-z plane.

3. (10 pts.) Let $v=\widehat{\imath}-\widehat{\jmath}$. Let $f\colon \mathbf{R}^3\to\mathbf{R}$ be defined by $f(u)=\mathrm{comp}_v(u)=u\cdot v/\,|v|$.
   (a) Find the values of f on the standard basis vectors of $\mathbf{R}^3$.
   (b) Is f is a linear map? Explain.

4. (10 pts.) Let $g\colon \mathbf{R}^2\to\mathbf{R}^2$ be the rotation by $\pi/2$ with respect to the origin.
   (a) Find the matrix that represents g with respect to the standard basis.
   (b) Write down the formula for g.

5. (extra credit) Sketch the following parametrized manifolds in $\mathbf{R}^3$.
   (a) $\cos t\widehat{\imath}+\sin t\widehat{\jmath}+t\widehat{k}$, where $0\le t<\infty$.
   (b) $\sin\varphi(\cos\theta\widehat{\imath}+\sin\theta\widehat{\jmath})+\cos\varphi\widehat{k}$, where $0\le\varphi\le\pi/2$, $-\pi<\theta\le\pi$.
   (c) $r(\cos\theta\widehat{\imath}+\sin\theta\widehat{\jmath})+z\widehat{k}$, where $1\le r\le 2$, $-\pi/2\le\theta\le\pi/2$, $0\le z\le 3$.

1	2	3	4	total (61)	%