Calculus for Applications / MAT 3243.001

Final / December 7, 1998 / Instructor: D. Gokhman

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Please show all work and box the answers.

1. (10 pts.) Let L be line $\{(3s+2, 4s+1)\colon s \in \mathbf{R}\}$ in $\mathbf{R}^2$. Express L as the locus of a single equation. Sketch the line.

2. (20 pts.) Sketch the following manifolds, express them in parametric form, and describe the boundary of each manifold:
   (a) The ray (half-line) in $\mathbf{R}^3$ from $\hat\imath$ in the direction $\hat\jmath - \hat k$.
   (b) Straight line segment in $\mathbf{R}^3$ from $\hat k$ to $\hat\jmath$.
   (c) Right half of the circle in $\mathbf{R}^2$ of radius 3 centered at $\hat\imath - 2\hat\jmath$.
   (d) Circle in $\mathbf{R}^3$ of radius 2 centered at $\hat\imath + \hat\jmath$ parallel to the y-z plane.
   (e) Parallelogram in $\mathbf{R}^3$ with 3 of the vertices $\hat k$, $2\hat k$, $\hat\imath + \hat\jmath$.

3. (10 pts.) Let $v = \hat\imath + \hat\jmath$, $w = -\hat\imath + \hat\jmath + \hat k$. Define $f\colon \mathbf{R}^3 \to \mathbf{R}^3$ by $f(u) = \operatorname{proj}_v(u) + \operatorname{proj}_w(u)$.
   (a) Find the values of f on the standard basis vectors of $\mathbf{R}^3$.
   (b) Is f is a linear map? Explain.

4. (10 pts.) Let $g\colon \mathbf{R}^2 \to \mathbf{R}^2$ be the rotation by $-3\pi/4$ with respect to the origin.
   (a) Find the matrix that represents g with respect to the standard basis.
   (b) Write down the formula for g.

5. (15 pts.) Let $f = y^{xz}$ and $F = (x+y)^3\hat\imath + \sin(xy)\hat\jmath + \cos(xyz)\hat k$.
   Compute $\operatorname{grad} f$, $\operatorname{div} F$, and $\operatorname{curl} F$.

6. (15 pts.) Compute $d\omega$
   (a) $\omega = y^2 \sin(xz)$
   (b) $\omega = xy\,dx + (x^2 - y^2)\,dy$
   (c) $\omega = x^3 z^2\,dy\,dz + \cos(2y)\,dz\,dx + e^x yz\,dx\,dy$

7. (10 pts.) Find an equation for the plane tangent to the surface given by $xe^z \cos y = 1$ at the point $-\hat\imath + \pi\hat\jmath$.

8. (20 pts.) Evaluate the following integrals
   (a) $\int_M -y\,dx + x\,dy - \,dz$, where M is the curve $\left\{\cos t\,\hat\imath + \sin t\,\hat\jmath + 2t\,\hat k\colon \pi \le t \le 2\pi\right\}$
   (b) $\int_M x\,dy\,dz + y\,dz\,dx$, where M is the cylinder $x^2 + y^2 = 9$, $0 \le z \le 2$
   (c) $\int_M z^2\,dx\,dy\,dz$, where M is the dowel $x^2 + y^2 \le 9$, $0 \le z \le 2$
   (d) Surface area: $\int_M |dS|$, where M is $\left\{s\hat\imath + (s+t)\hat\jmath + t\hat k\colon 0 \le s \le 2, 0 \le t \le 2\right\}$

1	2	3	4	5	6	7	8	total (110)	%