Calculus for Applications — MAT 3243, Fall 1997

Midterm, 1997/10/15 — Instructor: D. Gokhman

Name: ____________________

Please show all work and box the final answers.

1. (10 pts.) Let $u = (4, -3) \in \mathbf{R}^2$ and let $f\colon \mathbf{R}^2 \to \mathbf{R}$ be defined by $f(v) = \text{comp}_u v$. In other words, f maps a vector to its component along u. Compute df.
(Hint: let $v = (x, y)$ and express $f(v)$ as a function of x and y.)

2. (20 pts.) Consider the surface in $\mathbf{R}^3$ given by $(x^2 - z)(y + z^3)^4 = 1$. Find coefficients A, B, C, D such that $Ax + By + Cz = D$ gives the tangent plane to this surface at the point $(1, 1, 0)$.

3. (15 pts.) Parametrize the following curves. Specify the range for the parameter.
   (a) The straight line segment in $\mathbf{R}^2$ from $(2, -1)$ to $(6, 9)$.
   (b) The circle of radius 3 in the z-x plane centered at $(-1, 0, 5)$.
   (c) The graph of $y = \log x$ in the plane.

4. (20 pts.) Let $F\colon \mathbf{R}^3 \to \mathbf{R}^3$ be the vector field defined by $F(x, y, z) = \hat{k} + x\,\hat{\jmath} - y\,\hat{\imath}$. Integrate F along the helical segment $\mathbf{c}(t) = \cos(t)\,\hat{\imath} + \sin(t)\,\hat{\jmath} + t\,\hat{k}$, $0 \le t \le 2\pi$. What is the arclength of this segment?

5. (15 pts.) Let $F\colon \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $F(x, y, z) = ye^x\,\hat{\imath} + (e^x + ze^{yz})\,\hat{\jmath} + ye^{yz}\,\hat{k}$. Find $f\colon \mathbf{R}^3 \to \mathbf{R}$ such that $\text{grad}\, f = F$ and $f(0, 1, 0) = -1$.

1	2	3	4	5	total (80)	%