Calculus for Applications — MAT 3243, Fall 1997

Final, 1997/12/11 — Instructor: D. Gokhman

Name: ______________________ Pseudonym: ______________________

Please show all work.

1. (20 pts.) Find an equation for the tangent space to the manifold:
   (a) $x^2+y^2=5$ at $(1,-2)$ (b) $z=x^2+y^3$ at $(3,2,17)$
2. (30 pts.) For each vector field F compute DF, $\operatorname{curl} F$, and $\operatorname{div} F$:
   (a) $F=5xy\,\hat\imath+y^2\,\hat\jmath-x^2z\,\hat k$ (b) $F=z^2\,\hat\imath-xyz\,\hat k$
3. (40 pts.) For the following forms φ compute the exterior derivative $d\varphi$:
   (a) $\varphi=(x^2-z^3)\cos(3y-z^2)$
   (b) $\varphi=$ the component of (x,y,z) along $v=2\hat\imath-\hat\jmath-2\hat k$
   (c) $\varphi=xy^2\,dx+x^3y\,dy$
   (d) $\varphi=x\,dy\,dz+(z^3+y^2)\,dz\,dx$
4. (30 pts.) Parametrize each manifold and specify the range for the parameter(s).
   (a) The straight line segment from $(5,1,5)$ to $(7,8,9)$.
   (b) The plane through the points $(1,2,3)$, $(1,-2,3)$, and $(0,0,1)$.
   (c) The graph of $y=\ln(x)$ in the plane.
   (d) The graph of $z=\sin(x)+\cos(y)$.
   (e) The disk of radius 2 centered at $(7,8,9)$ parallel to the z-x plane.
   (f) The ball of radius 3 centered at $(5,1,5)$.
5. (20 pts.) For the following manifolds M: (i) Sketch M. (ii) Describe ∂M.
   (a) M as in 6(a). (b) M as in 6(b). (c) M as in 6(d). (d) M as in 6(e).
6. (60 pts.) Evaluate the integral $\int_M \varphi$.
   Hint: If $\varphi=d\omega$ or $M=\partial\Omega$, find ω or Ω, and use FTC: $\int_\Omega d\omega=\int_{\partial\Omega}\omega$.
   (a) $\varphi=xy\,dx-y^2\,dy+x\,dz$, $M=\{\cos(t)\,\hat\imath+\sin(t)\,\hat\jmath\colon \pi\le t\le 2\pi\}$.
   (b) $\varphi=x\,dy-y\,dx+4\,dz$, $M=\left\{\cos(t)\,\hat\imath+\sin(t)\,\hat\jmath+2t\,\hat k\colon 0\le t\le 2\pi\right\}$.
   (c) $\varphi=3x^2y\,dx+(x^3+2yz)\,dy+y^2\,dz$, $M=$ the segment $(7,8,9)\to(9,9,-9)$
   (d) $\varphi=5y\,dx-x\,dy+3z^2\,dz$, $M=\{(x,y,z)\colon x^2+y^2=1, z=0\}$.
   (e) $\varphi=x\,dy\,dz+y\,dz\,dx+z^2\,dx\,dy$,
   $M=\left\{2\cos(\theta)\,\hat\imath+2\sin(\theta)\,\hat\jmath+z\,\hat k\colon 0\le\theta\le 2\pi, -3\le z\le 3\right\}$
   (f) $\varphi=(3x+z^y)\,dy\,dz-(z^4+1)^x\,dz\,dx$, $M=\{(x,y,z)\colon (x-1)^2+y^2+z^2=4\}$.

Have a great break! –*D*

1	2	3	4	5	6	Total (200)